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Modelling of cyclic fatigue stress for life prediction of structural ceramics

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The stress models of a cyclic fatigue test for structural ceramics were developed using the theory of fracture mechanics to predict the accurate life cycles of the specimens. Four kinds of models were tried to obtain the representative stresses corresponding to the cyclic stresses applied to the alumina specimens. Crack-growth exponents of 21.81, 22.15, 24.57 and 24.43 were obtained from the arithmetic mean stress model, the integrated stress model, the maximum stress model, and the equivalent static stress model, respectively. It is considered that the equivalent static stress model is the most reasonable and gives the most accurate value of the crack-growth exponent from the view point of the theoretical background for developing the model.

1. Introduction

The development of a technique to evaluate the reliability of sintered ceramic bodies is as important as that of powder preparation, fabrication of ceramic parts and other ceramic processing techniques for the applications of ceramics to structural and mechanical uses. It is well known that fracture of ceramics is caused by the growth of microcracks existing on the surfaces or inside the ceramic parts, even under a working load much less than the fracture stress. The fatigue life of ceramics is controlled by the subcritical crack growth [1] and the crack-growth rate, V, is expressed as

$$V = Ak_{\mathrm{I}}^{n} \tag{1}$$

where K_{I} is the stress intensity factor, and A and n are material constants. Because the crack-growth exponent, n, of most ceramics has a value higher than 20, ceramics generally follow a catastrophic failure by rapid crack growth [2–5] as can be expected from Equation 1. The fatigue lifetime of ceramics can be expressed by Equation 2, derived on the basis of Griffith's fracture criterion [6]

$$\left(\frac{\sigma_1}{\sigma_2}\right)^{n} = \left(\frac{t_2}{t_1}\right) \tag{2}$$

where t_1 is the time to failure of the ceramics under an applied stress σ_1, and t_2 is the time to failure under an applied stress σ_2.

The applied stress is constant in static fatigue; however, it varies with time in cyclic fatigue. Therefore, it is desirable to derive an appropriate representative "static" stress corresponding to the cyclic stress applied to the alumina ceramics, which should exactly reflect the growth of the microcrack existing in alumina ceramics under cyclic stress.

In this study the fatigue life of alumina ceramics was measured under cyclic stress loading and the stresses in Equation 2 were studied to predict the lifetime of the alumina ceramics under cyclic stressing.

2. Theoretical background

The fatigue lifetime of ceramics can be estimated from the correlation between the two equations related to slow crack growth and stress intensity factor, as shown in Equation 3

$$V = \frac{\mathrm{d}a}{\mathrm{d}t} =$$

$$AK_{\mathrm{I}}^{n} = A(Y\sigma a^{1/2})^{n} \tag{3}$$

where σ is the applied stress, a the crack size, t the time, A and n the material constants, K_{I} the stress intensity factor, and Y the shape factor.

Integration of Equation 3 to the final failure time gives

$$\int_{a_{\mathrm{i}}}^{a_{\mathrm{f}}} a^{-n/2}\mathrm{d}a = AY^{n}\int_{0}^{t}\sigma^{n}\mathrm{d}t \tag{4}$$

$$a_{\mathrm{f}}^{(2-n)/2} - a_{\mathrm{i}}^{(2-n)/2} = \frac{2-n}{2}AY^{n}\sigma^{n}t \tag{5}$$

Because $a_{\mathrm{f}} \gg a_{\mathrm{i}}$ and $n \gg 1$, Equation 6 can be obtained

$$\frac{2}{(n-2)AY^{n}a_{\mathrm{i}}^{(n-2)/2}} = \sigma^{n}t \tag{6}$$

The left-hand side of Equation 6 can be represented by a constant c, and so Equations 7 and 8 can be derived

$$c = \frac{2}{(n-2)AY^{n}a_{\mathrm{i}}^{(n-2)/2}} \tag{7}$$

$$c = \sigma_1^{n}t_1$$
$$= \sigma_2^{n}t_2 \tag{8}$$

Equation 2 thus has been derived. Plotting $\log \sigma$ versus $\log t$ using Equation 2 gives the slope n, and hence the time to failure can be predicted without actual loading.

To apply Equation 2 to the prediction of fatigue life, it is necessary to express a representative stress corresponding to the cyclic stress applied to the specimen. Four possible models for the representative stress will be discussed.

2.1. Arithmetic mean stress model

The arithmetic mean value of the maximum and minimum stresses in a single cycle of the repeated cyclic stressing is defined as a corresponding representative stress, as shown in Equation 9

$$\sigma_{\text{mean}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} \quad (9)$$

where σ_{mean} is the arithmetic mean stress, $\sigma_{\max}$ the maximum stress, and $\sigma_{\min}$ the minimum stress.

2.2. Integrated stress

Integration of cyclic stress during stressing time can be done through integrating the area represented by the cyclic stress curve along the time elapsing until failure occurs, as shown in Equation 10

$$\sigma_{\text{int}} = t^{-1} \int_0^t \sigma(t) \mathrm{d}t \quad (10)$$

Equation 10 defines the integrated stress which is also one of the representative stresses, where σ_{int} is the integrated stress, $\sigma(t)$ the applied stress, and t the time to failure.

2.3. Maximum stress model

The maximum stress in a cycle is defined as a representative stress, which has already been used to predict the fatigue lifetime [2–5, 9].

2.4. Equivalent static stress model

Equations 11 and 12 can be derived by letting the cyclic stress be equal to the equivalent static stress, σ_{es}, as a static stress, as given in Equation 6

$$\sigma_{\text{es}} = \left[t^{-1} \int_0^t [\sigma(t)]^n \mathrm{d}t \right]^{1/n} \quad (11)$$

$$\int_0^t [\sigma(t)]^n \mathrm{d}t = \sigma_{\text{es}}^n t \quad (12)$$

Applying Equation 12 to Equations 6 and 7 gives Equation 13

$$\sigma_{\text{es}}^n t = c \quad (13)$$

All the above-mentioned representative stresses, σ_{mean}, σ_{int}, $\sigma_{\max}$ and σ_{es} can be applied to predict the fatigue life of alumina ceramics under cyclic stressing.

3. Experiments

The rectangular-type alumina specimens of 3 mm × 4 mm × 40 mm were prepared by cutting, grinding and polishing after they were sintered at 1670 °C for 2 h. The properties of the alumina specimens are given in Table I. The cyclic fatigue test machine for this investigation was constructed as schematically shown in Fig. 1.

The cyclic fatigue test was conducted by stressing the alumina specimen on a three-point bend tester with a frequency of 0.5 Hz. The stresses applied to the specimens were in the range of 250–350 MPa, calculated using

$$\sigma = \frac{3Pl}{2bd^2} \quad (14)$$

where P is the load, l the span, b the width, and d the thickness of the specimen.

A computer program was developed to calculate the equivalent static stress and the crack-growth exponent in this investigation and the flowchart of the program is given in Fig. 2.

TABLE I Properties of the alumina specimen

ρ	3900 kg m^{-3} (97.9%)
σ	360–420 MPa
K_{IC}	3.6–4.5 MPa m$^{1/2}$
Grain size	2–10 μm (mean 5 μm)
H_v	11.5–13 GPa

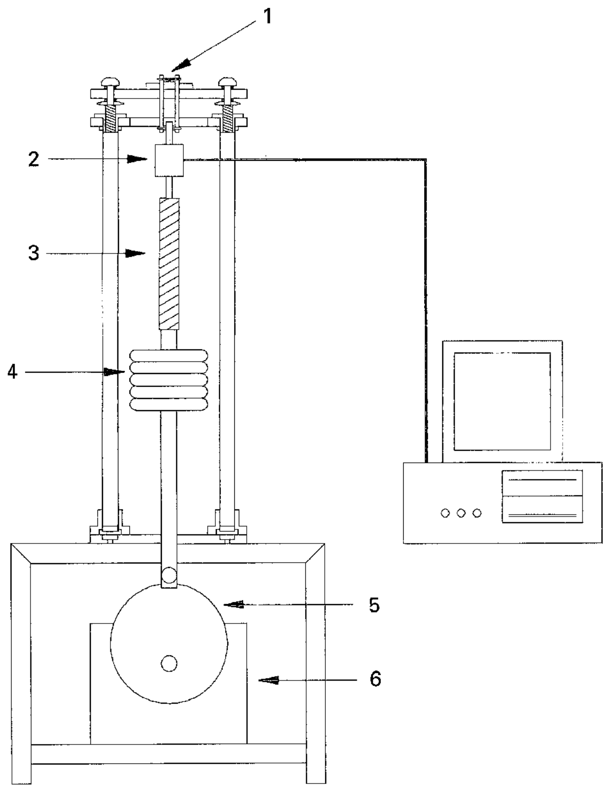

Figure 1 Schematic diagram of the cyclic fatigue machine. 1, Specimen; 2, load cell; 3, spring; 4, weight; 5, cam; 6, motor.

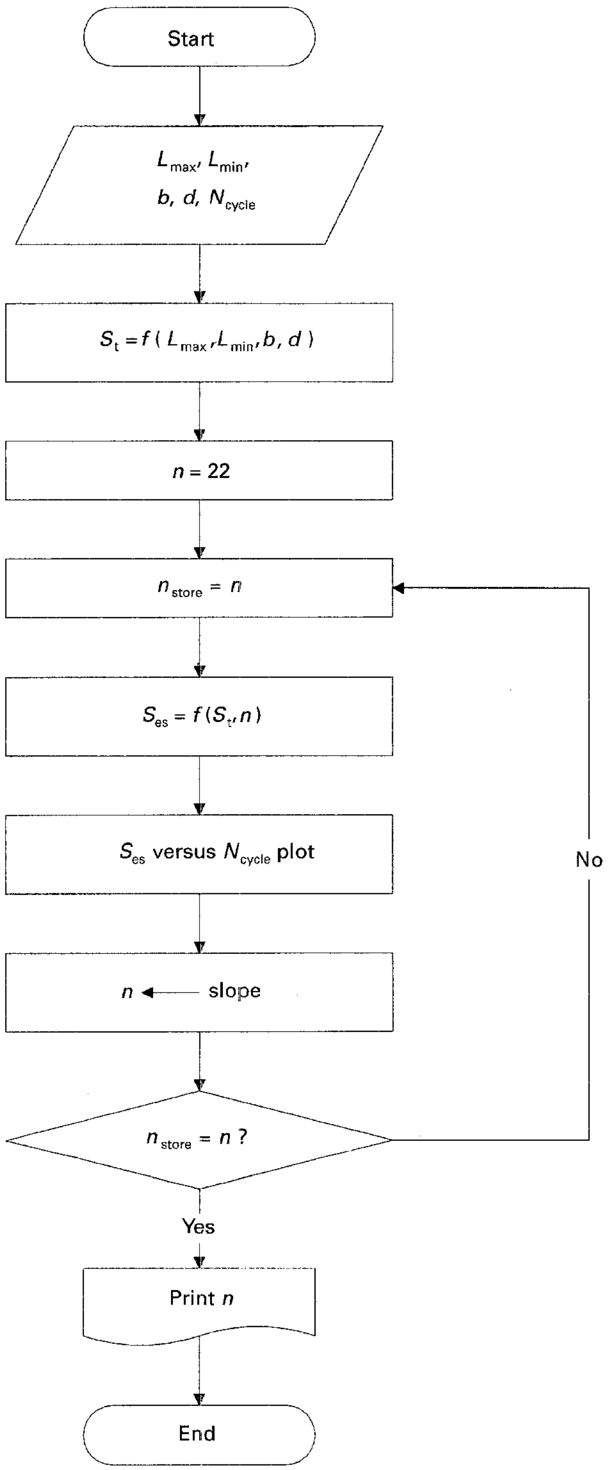

Figure 2 Flowchart of the calculation program for *n*.

4. Results and discussion

The variation of the load applied to the specimens with time is shown in Fig. 3. The wave pattern shows a kind of the sine curve with a frequency of 0.5 Hz. Cyclic fatigue according to the pattern of the sine curve was applied to the specimens. The four kinds of representative stress models developed in this study were demonstrated graphically and their accuracies were compared.

4.1. Arithmetic mean stress model and integrated stress model

We assume that two wave functions, A and B, have the same wavelength, π, and frequency, 0.5 Hz, but different amplitudes, 100 and 50, for A and B, respectively, as shown in Fig. 4. In this case the arithmetic mean stress, σ_{mean}, and the integrated stress, σ_{int} are the same, 200 MPa, for both functions A and B, as can be seen in Fig. 4. However, it is expected that they will have different fatigue lifetimes because the two wave

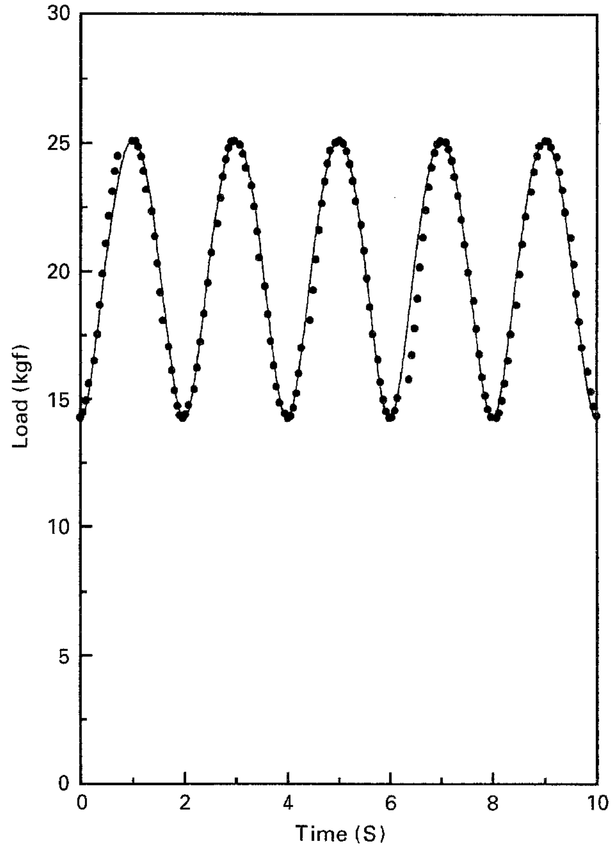

Figure 3 Typical load–time profile for cyclic fatigue loading.

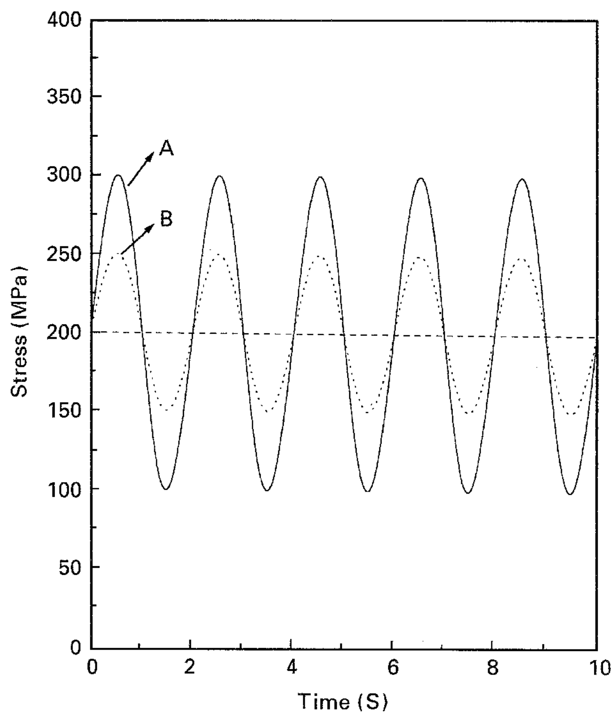

Figure 4 Schematic drawing of the loading wave forms for the cyclic fatigue test. A, $\sigma(t) = 100\sin(\pi t) + 200$; B, $\sigma(t) = 50\sin(\pi t) + 200$.

functions have different maximum and minimum stresses. This means that the arithmetic mean stress model and the integrated stress model do not correctly reflect the representative stress corresponding to the applied cyclic stress. It is also considered that the arithmetic mean stress and integrated stress cannot be reasonably applied for lifetime prediction.

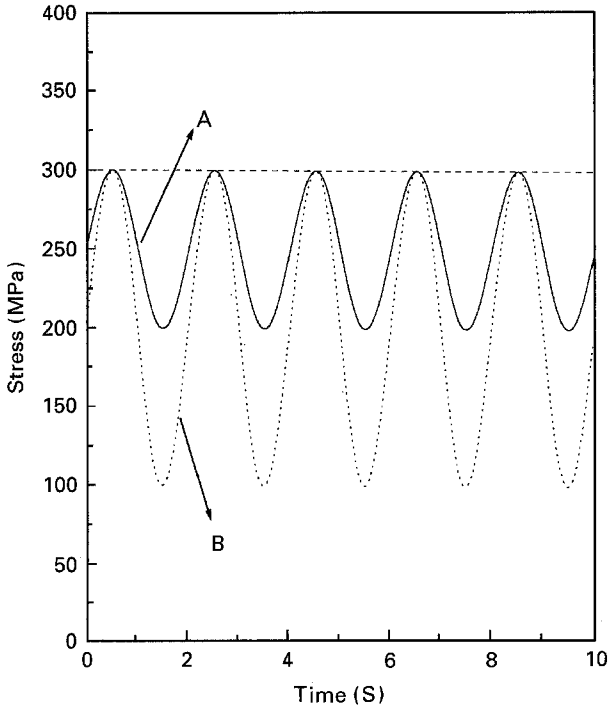

Figure 5 Schematic drawing of the loading wave forms for the cyclic fatigue test. A, $\sigma(t) = 50\sin(\pi t) + 250$; B, $\sigma(t) = 100\sin(\pi t) + 200$.

4.2. Maximum stress model and equivalent static stress model

The maximum stress model gives the maximum stress, $\sigma_{max} = 300$ MPa, in Fig. 4, and the equivalent static stress model gives the equivalent static stress, $\sigma_{es} = 276.79$ MPa, from Equation 11 for the wave function A. In the same way, the maximum stress, σ_{max}, and the equivalent static stress, σ_{es}, can be determined as 250 and 233.19 MPa, respectively, for the wave function B. Both the maximum stress model and the equivalent static stress model are considered to be quite reasonable. However, the maximum stress model does not reflect the minimum stress when the wave function shows different minimum stress, as shown in Fig. 5. The maximum stress model gives the same maximum stress, 300 MPa, for both functions A and B, as shown in Fig. 5, however, the equivalent static stress model reflects the different minimum values by showing 280.95 MPa for function A and 276.79 MPa for function B, as shown in Fig. 5. In fact, the maximum stress model has already been accepted and generally used as the representative stress to predict the fatigue lifetime of ceramics [2–5, 9] via approximation by neglecting tedious integration. The equivalent static stress model has been developed theoretically in this study on the basis of fracture mechanics without neglecting the integration. Therefore, it can be said that the equivalent static stress model is more reasonable and reliable than the maximum stress model to predict the fatigue lifetime of ceramics.

The representative stresses of the four models were calculated and plotted against time (or number of cycles) in Figs 6–9. The data of the arithmetic mean stress are plotted against time in Fig. 6, those of the integrated stress against time are given in Fig. 7. Fig. 8 shows the maximum stress plotted against time, while

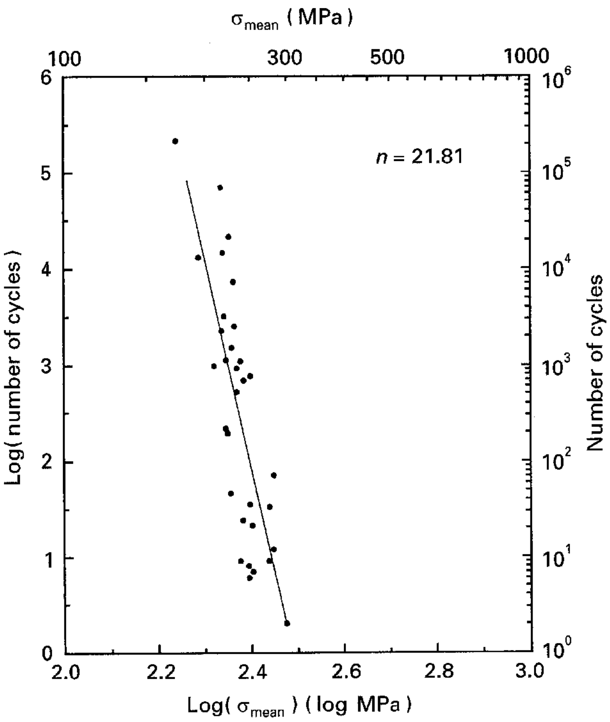

Figure 6 Double logarithmic plot of the arithmetic mean stress against number of cycles.

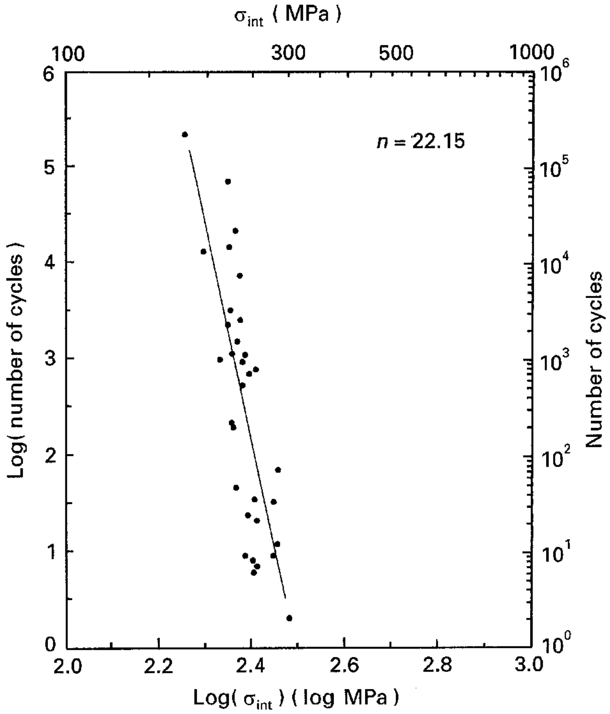

Figure 7 Double logarithmic plot of the integrated stress against number of cycles.

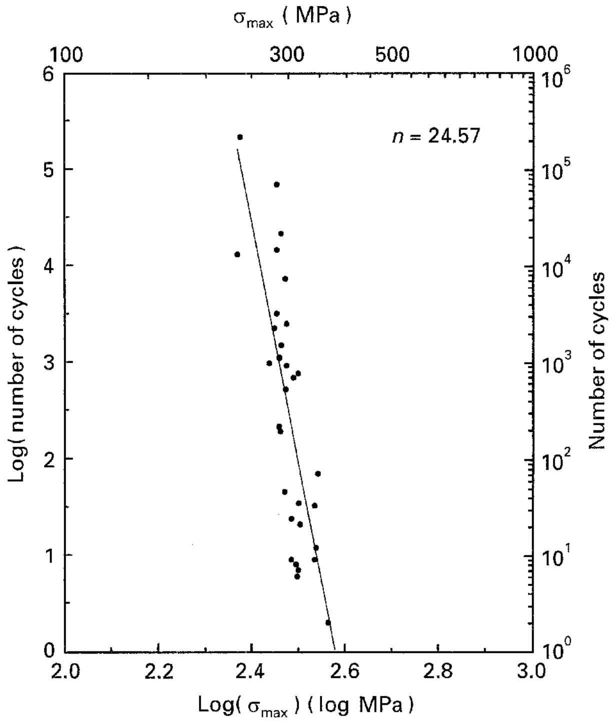

Figure 8 Double logarithmic plot of the maximum stress against number of cycles.

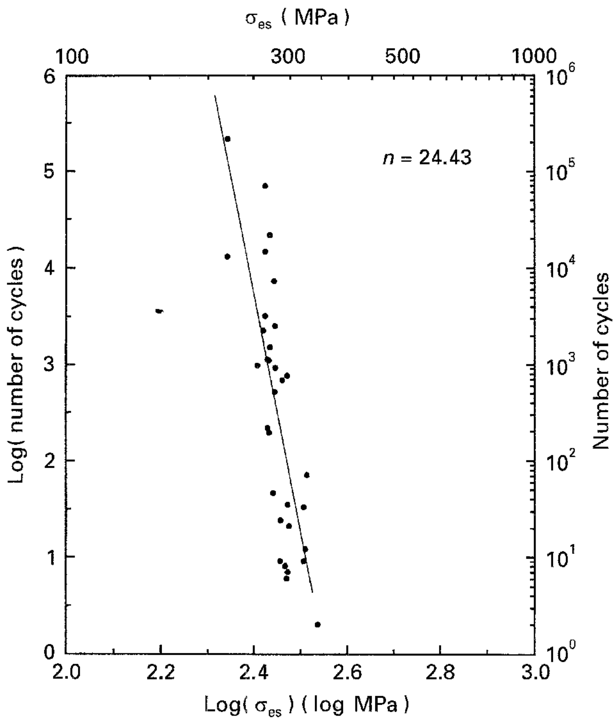

Figure 9 Double logarithmic plot of the equivalent static stress against number of cycles.

the variation of equivalent static stress with time is shown in Fig. 9. The crack-growth exponent, n, for each model can be obtained from the slopes of log σ versus log t plots. The crack-growth exponents, n, for the arithmetic mean stress model, the integrated stress model, the maximum stress model, and the equivalent static stress model are 21.81, 22.15, 24.57, and 24.43, respectively, obtained by the least-squares method. The maximum stress model shows a value very close to that of the equivalent static stress model.

5. Conclusion

Four kinds of model were suggested to obtain the most accurate representative static stress corresponding to the cyclic stress applied to the alumina specimens to predict the lifetime of alumina ceramics under cyclic fatigue. The arithmetic mean stress model gives a crack-growth exponent of 21.81, the integrated stress model gives 22.15, the maximum stress model gives 24.57, and the equivalent static stress model gives 24.43. It is considered that the equivalent static stress model is the most reasonable one, and gives the most accurate and reasonable crack-growth exponent value.

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